Comment on "Diffusive Separation in Shock Waves and Freejets of Nitrogen-Helium Mixtures"

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IN a recent article Sebacher[1] has described some experimental measurements on the separation of mixtures in the unconfined expansion from a supersonic nozzle as well as in shock waves created in this flowfield. This flow should not be confused with that resulting from the unconfined expansion from a sonic orifice which is usually referred to as a freejet. Thus, although one would expect to observe a monotonically decreasing concentration of the lightest species on the centerline of a freejet[2,3] the opposite result is reported in Ref. 1. The interpretation of the latter result requires some detail about the nozzle flowfield and it would have been of interest to have had available some experimental data on the pressure distribution in the flow as well as the stagnation conditions. A measurement of the centerline variation of stagnation pressure for both monatomic and diatomic gases would have allowed at least a theoretical estimate[2] of the diffusive separation on the nozzle axis.

With regard to the discussion on the shock-wave results, there is no justification for applying the Rankine-Hugoniot relations individually to each component of the mixture. The shock-wave relations should have been applied to a gas mixture with the appropriate thermodynamic properties and equation of state.[4,5] It is evident from consideration of mass conservation that the species density ratio across a plane normal shock must be the same for all species.[5,6] Sebacher's experimental result that the density ratios were not the same can be explained by shock-wave curvature, relative to the flow streamlines, with a radius of curvature not much greater than the shock thickness. The complete lack of experimental data on the flow variables upstream of the shock wave prevents the reader from making any estimate of the shock thickness variation with the initial mixture concentration as well as with the theoretical results for pure helium and nitrogen.

Previous electron beam experiments[7,8] have indicated that the rotational distribution function through a shock wave cannot be represented by a single temperature for high Mach number flows of the order of 10 or more. Sebacher does not present any results of his intensity measurements of rotational structure. If his results also indicated an apparent non-Boltzman distribution, then it would be interesting to learn how he has interpreted his measurements to yield temperature profiles.

One further comment might be made. Sebacher has derived the ratio of the collision cross sections for direct excitation of the selected transitions in helium and nitrogen from a measurement of the intensity ratio at a known concentration. This result is incorrect because of the assumption of equal constants in Eqs. (9) and (10) for the number density of excited particles in each species. In the absence of collision quenching, the number density of directly excited particles is proportional to the lifetime of the excited state[9] as well as to the other terms indicated by Sebacher. As a result the intensity ratio, Eq. (11) should be modified by the ratio of the appropriate lifetimes for the excited states.

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Reply by Author to R. E. Center

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IN a search of the literature, the author has not found the term freejet to be restricted only to sonic orifices as indicated in the preceding Comment, but the term has also been applied to supersonic nozzles exhausting into a low pressure region.[1] Aside from this point, the author agrees with R. E. Center that the supersonic ($M = 4.42$) nozzle selected was inappropriate for a comparison with Sherman's theory,[2] since the enrichment of the heavier particles along the centerline obviously has taken place inside the nozzle where it could not be measured with the existing electron beam apparatus. This nozzle was selected because of the quality of the normal shock wave which could be obtained. The study of diffusion in the shock region was the primary objective of this study. After the *AIAA Journal* article was published, an additional investigation on sonic orifices was conducted. The results of this second experiment were in general agreement with the results shown in the subject paper and these data will be published in the near future.

Regarding the shock-wave discussion, the author did not mean to imply that the Rankine-Hugoniot relations could be applied individually to each component, but these relations were introduced to show the density ratio range for pure gases at the given Mach number. The shock wave studied was different from that studied by Rothe[3] and Center,[4] in that in this investigation, the shock wave was a Mach disk which should be normal to the streamline in the region of the centerline. The measurement of the flow properties upstream of this shock with other than the electron beam would be difficult because any other probe would move the Mach disk and change the parameter being measured. Only electron beam data are presented in the paper. The stream conditions both upstream and downstream of this shock are in a rarefied region, and this fact may also contribute to the increase in the N_2 concentration downstream of the shock, since the He may diffuse in both directions and should not necessarily be attributed to shock curvature. Since Sherman's theory[5] is based on continuum calculations, one should be cautious in comparing the data with the theory. Furthermore, in a mix-

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ture of N_2 and He where separation has taken place, the shock wave contains gradients in both concentrations and in γ so that the shock wave relations are difficult to apply.

In measuring the rotational temperatures through the shock wave, the author has also found non-Boltzmann distributions, as have all electron beam investigators. The temperatures presented are approximated by merging the rotational distribution functions. There is a considerable controversy over the analysis of rotational temperatures (see Ref. 6) about which the author has little to offer at the present time.

Finally, the statement by R. E. Center that Eq. (11) should be modified by a ratio of the appropriate lifetimes for the excited states is an oversight on his part since this equation contains the ratio of Einstein's transition probabilities of emission (A_{nm} of He to N_2) which are directly related to the lifetimes by

$$\frac{1}{\tau} = \sum_m A_{nm}$$

This relationship is discussed in Ref. 7.

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Comments on Schlieren Measurements of the Inviscid Hypersonic Wake of a Sphere

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WILSON[1] has found that a sphere at high Mach number produces a well-defined trail 8 diam in width for thousands of diameters behind the body in a schlieren photograph. This phenomenon appears to be independent of Reynolds number and body material and is called the "inviscid wake" by Wilson. He finds the "edge" of this wake by finding the location where the film contrast is greatest and identifies this location as the place where the flowfield "density gradient was greatest." If this statement is modified to read: "the radial location on the schlieren photo where the integrated density gradient along the light path was greatest," then it is interesting to see if this straightforward interpretation of the inviscid wake seen on the schlieren agrees quantitatively with the expected "inviscid wake width."

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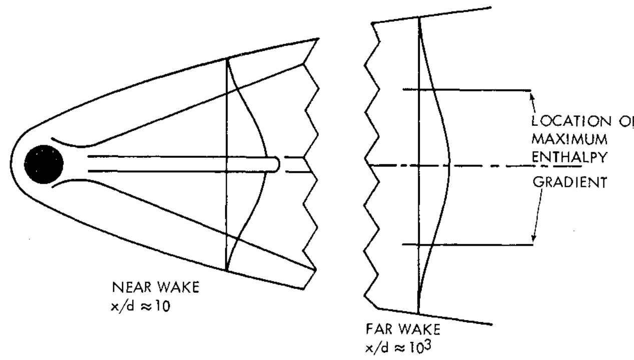

Fig. 1 Typical enthalpy profiles in near and far wake of a sphere.

Figure 1 shows typical enthalpy profiles expected in the near and far wake of a sphere for Reynolds numbers at which Wilson's data were taken. The inner "viscous wake" near the body is smeared out by x/d of order 10^3 but the shock-produced inviscid wake continues for several thousand diameters, its characteristic diffusion time being longer owing to its larger size.

Denoting the distance from the axis on a schlieren photograph by y, the film contrast in a schlieren system at y is proportional to the light deflection,[2] which is given by

$$\epsilon(y) = 2y \int_y^\infty \frac{(\partial n/\partial r)dr}{(r^2 - y^2)^{1/2}} \quad (1)$$

where r is the radial coordinate of the axisymmetric wake and n is the index of refraction of the wake. Since the wake electron density is much smaller than the neutral density, detectable variations in n are due only to neutral density variations and $n \sim \rho/\rho_\infty$.

We will attempt to evaluate (1) for a very simple wake model. The drag on the body is

$$D = 2\pi \int_0^\infty \rho u(u_\infty - u)rdr \quad (2)$$

at large x/d. Further at this distance $u \approx u_\infty$, the total enthalpy is approximately uniform, $H \approx H_\infty$, and so is the pressure $p \approx p_\infty$, so that $u(u_\infty - u) \approx h - h_\infty$ and $\rho/\rho_\infty \approx$

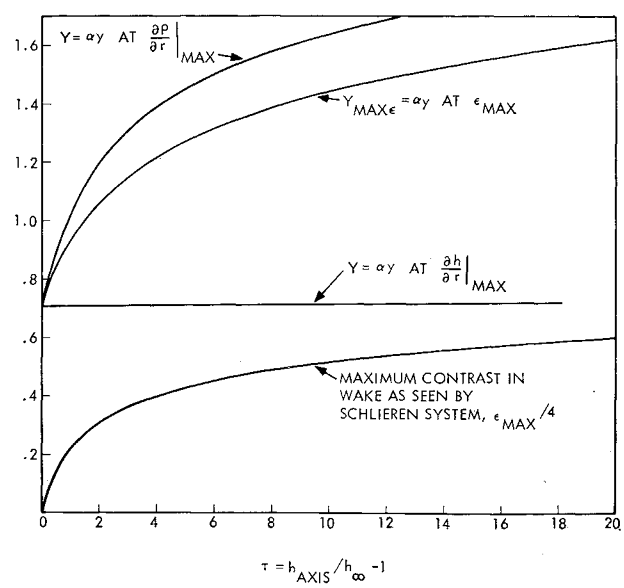

Fig. 2 Values of Y for maximum light deflection, enthalpy gradient, and density gradient. Also maximum schlieren light deflection $\epsilon/4$, vs $\tau = h_A/h_\infty - 1$. Gaussian enthalpy profile, $h/h_\infty = 1 + \tau \exp - Y^2$ is assumed.